DAY THIRTY FOUR

Radioactivity

Learning & Revision for the Day

- Law of Radioactive Decay
- Nuclear Fission
- Nuclear Fusion
- Mass Defect and Binding Energy

Radioactivity is the phenomenon of spontaneous emission of radiations by heavier nucleus. Some naturally occurring radioactive substances are uranium, thorium, polonium, radium, neptunium, etc. In fact, all elements having atomic number $Z > 82$ are radioactive in nature.

Radiations emitted by radioactive substances are of three types, namely (i) α-particles, (ii) β-particles and (iii) γ- rays.

- **α-particles** are positively charged particles with charge $q_\alpha = +2e$ and mass $m_\alpha = 4m_p$. Thus, α-particles may be considered as helium nuclei (or doubly charged helium ions). Ionising power of α-particles is maximum, but their penetrating power is minimum.
- **β-particles** are negatively charged particles with rest mass as well as charge same as that of electrons. But origin of β-particles is from the nucleus. Their ionising power is lesser than that of α-particles, but speed as well as penetrating power is much greater than that of α-particles. Generally, β-decay means β^--decay.
- **γ-rays** are electromagnetic radiations of extremely short wavelengths. Thus, γ-rays travel with the speed of light. Their ionising power is least, but penetrating power is extremely high. These are not deflected either in an electric or a magnetic field.

Law of Radioactive Decay

According to Rutherford-Soddy's law for radioactive decay, 'The rate of decay of a radioactive material at any instant is proportional to the quantity of that material actually present at that time.'

Mathematically, $\frac{dN}{dt} \propto N$ or $\frac{dN}{dt} = -\lambda N$

Here, λ is a proportionality constant, known as the **decay constant** (or disintegration constant). Unit of λ is s^{-1} or day^{-1} or $year^{-1}$, etc.

It can be shown that number of nuclei present after time t is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = number of nuclei present at time $t = 0$.

Again, number of nuclei decayed in time t will be

$$N - N_0 = N_0 [e^{-\lambda t} - 1]$$

$=$ number of **daughter nuclei** produced at time t.

Half-Life Period ($T_{1/2}$)

It is the time in which, activity of the sample falls to one-half of its initial value.

Thus, for $t = \frac{T}{2}, N = \frac{N_0}{2}$ and $R = \frac{R_0}{2}$

- The half-life period is related to decay constant λ as

$$T_{1/2} = \frac{0.693}{\lambda}$$

- After n half-lives, the quantity of a radioactive substance left intact (undecayed) is given by

$$N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

Mean Life Period (τ)

- Mean life of a radioactive sample is the time, at which both N and R have been reduced to $\frac{1}{e}$ or e^{-1} or 36.8% of their initial values. It is found that $\tau = \frac{1}{\lambda}$.
- Half-life $T_{1/2}$ and mean life τ of a radioactive sample are correlated as, $T_{1/2} = 0.693\,\tau$ or $\tau = 1.44 T_{1/2}$.

Activity

The activity of a radioactive substance is defined as the rate of disintegration (or the count rate) of that substance.

Mathematically, activity is defined as

$$R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0\, e^{-\lambda t}$$

where, $R_0 = \lambda N_0 =$ initial value of activity.

Units of activity are

- 1 becquerel = Bq = 1 disintegration per second (SI unit)
- 1 curie = 1 Ci = 3.7×10^{10} Bq
- 1 rutherford = 1 Rd = 10^6 Bq

Mass Energy Relation

In nuclear physics, mass is measured in **unified atomic mass units** (u), 1 u being one-twelfth of the mass of carbon-12 atom and equals 1.66×10^{-27} kg. It can readily be shown using $E = mc^2$ that, 1 u mass has energy 931.5 MeV

Thus, $1\text{ u} = 931.5\text{ MeV}$ or $c^2 = 931.5\text{ MeV/u}$

A unit of energy may therefore be considered to be a unit of mass. For example, the electron has a rest mass of about 0.5 MeV.

If the principle of conservation of energy is to hold for nuclear reactions it is clear that mass and energy must be regarded as equivalent. The implication of $E = mc^2$ is that any reaction producing an appreciable mass decrease is a possible source of energy.

- At the rest, mass energy of each of electron and positron, is

$$E_0 = m_0 c^2 = 9.1 \times 10^{-31} \times (3 \times 10^8)^2\text{ J}$$
$$= 0.51\text{ MeV}$$

Therefore, an energy of atleast 1.02 MeV is needed for pair production.

Mass Defect and Binding Energy

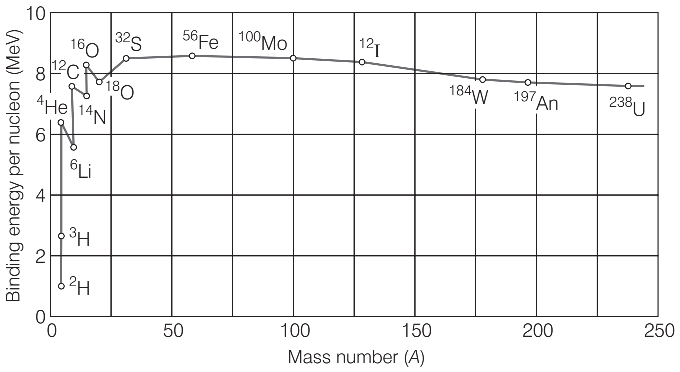
- The difference in mass of a nucleus and its constituent nucleons is called the mass defect of that nucleus. Thus, Mass defect, $\Delta M = Zm_p + (A - Z)m_n - M$ where, M is the mass of a given nucleus.
- **Packing fraction** of an atom is the difference between mass of nucleus and its mass number per nucleon. Thus, Packing fraction $= \frac{M - A}{A}$.
- The energy equivalent of the mass defect of a nucleus is called its **binding energy.**
Thus, binding energy, $\Delta E_b = \Delta M c^2$
$= [Zm_p + (A - Z)m_n - M]\, c^2$

If masses are expressed in atomic mass units, then
$\Delta E_b = \Delta M \times 931.5$ MeV
$= [Zm_p + (A - Z)m_n - M] \times 931.5$ MeV

- **Binding energy per nucleon** (ΔE_{bn}) is the average energy needed to separate a nucleus into its individual nucleons.

Thus, $\Delta E_{bn} = \frac{\Delta E_b}{A} = \frac{\Delta M \times 931}{A} \frac{\text{MeV}}{\text{Nucleon}}$

- The figure show binding energy per nucleon *versus* mass number. The nuclides showing binding energy per nucleon greater than 7.5 MeV/nucleon are stable.



Binding energy per nucleon *versus* mass number variation

NOTE
- Nucleons attract each other when they are separated by a distance of 10^{-14} m.
- The density of nucleus is of the order of 10^{17}.

Nuclear Fission

Nuclear fission is the process of splitting of a heavy nucleus ($^{235}_{92}U$ or $^{239}_{94}Pu$) into two lighter nuclei of comparable masses along with the release of a large amount of energy ($\simeq 200$ MeV) after bombardment by slow neutrons.

A characteristic nuclear fission reaction equation for $^{235}_{92}U$ is

$$^{1}_{0}n(\text{slow}) + ^{235}_{92}U \longrightarrow ^{236}_{92}U \longrightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3\ ^{1}_{0}n + Q$$

In the fission of uranium, the percentage of mass converted into energy is about 0.1%.

Controlled Chain Reaction and Nuclear Reactor

- In the fission of one nucleus of $^{235}_{92}U$, on an average, $2\frac{1}{2}$ neutrons are released. These released neutrons may further, trigger more fissions causing more neutrons being formed, which in turn may cause more fission. Thus, a self sustained nuclear chain reaction is formed. To maintain the nuclear chain reaction at a steady (sustained) level, the extra neutrons produced, are absorbed by suitable neutron absorbents like cadmium or boron.
- Neutrons formed as a result of fission have an energy of about 2 MeV, whereas for causing further fission, we need slow thermal neutrons having an energy of about 0.3 eV. For this purpose, suitable material called a **moderator** is used, which slow down the neutrons. Water, heavy water and graphite are commonly used as moderators.
- A **nuclear reactor** is an arrangement in which nuclear fission can be carried out through a sustained and a controlled chain reaction and can be employed for producing electrical power, for producing different isotopes and for various other uses.
- Power of a reactor, $P = \frac{nE}{t}$, where n = number of atoms undergoes fission in time t seconds and E = energy released in each fission.

Reproduction Factor

Reproduction factor (k) of a nuclear chain reaction is defined as

$$k = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss / Absorption of neutrons}}$$

- If $k = 1$, then the chain reaction will be steady and the reactor is said to be **critical**.
- If $k > 1$, then the chain reaction is accelerated and it may cause explosion in the reactor. Such a reactor is called **super-critical**.
- If $k < 1$, then chain reaction gradually slows down and comes to a halt. Such a reactor is called **sub-critical**.

The reactors giving fresh nuclear fuel which often exceeds the nuclear fuel used is known as **breeder reactor**.

Nuclear Fusion

Nuclear fusion is the process, in which two or more light nuclei combine to form a single large nucleus.

The mass of the single nucleus, so formed is less than the sum of the masses of parent nuclei and this difference in mass, results in the release of tremendously large amount of energy.

The fusion reaction going on in the central core of sun is a multistep process, but the net reaction is

$$4\ ^{1}_{1}H + 2\varepsilon^{-} \longrightarrow ^{4}_{2}He + 2\nu + 6\gamma + 26.7 \text{ MeV}$$

When two positively charged particles (protons or deuterons) combine to form a larger nucleus, the process is hindered by the Coulombian repulsion between them.

To overcome the Coulombian repulsion, the charged particles are to be given an energy of atleast 400 keV.

For this, proton/deuterons must be heated to a temperature of about 3×10^{9} K.

Nuclear fusion reaction is therefore, known as **thermo nuclear fusion** reaction.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 An electron of 1.02 MeV and a positron of 1.02 MeV collide and annihilate into energy producing two γ-photons. The energy of each γ-photon will be

(a) 1.02 MeV (b) 2.04 MeV
(c) 0.51 MeV (d) 1.53 MeV

2 When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom

(a) do not change for any type of radioactivity
(b) change for α and β-radioactivity, but not for γ-radioactivity
(c) change for α-radioactivity, but not for others
(d) change for β-radioactivity, but not for others

3 A radon nucleus $_{86}Rn^{222}$ of mass 3.6×10^{-25} kg undergoes α-decay. α-particle has mass 6.7×10^{-27} kg and energy 8.8×10^{-13} J. The resulting nucleus is

(a) $_{84}Sr^{220}$ (b) $_{84}Po^{218}$
(c) $_{84}Sn^{220}$ (d) None of these

4 A nucleus $^{m}_{n}X$ emits one α-particle and two β-particles. The resulting nucleus is ➜ CBSE AIPMT 2011

(a) $^{m-6}_{n}Z$ (b) $^{m-4}_{n}X$ (c) $^{m-4}_{n-2}Y$ (d) $^{m-6}_{n-4}Z$

5 The number of beta particles emitted by a radioactive substance is twice the number of alpha particles emitted by it. The resulting daughter is an ➜ CBSE AIPMT 2009

(a) isobar of parent (b) isomer of parent
(c) isotone of parent (d) isotope of parent

6 Radioactive material A has decay constant 8λ and material B has decay constant λ. Initially, they have same number of nuclei. After what time, the ratio of number of nuclei of material B to that A will be $\frac{1}{e}$? ➜ NEET 2017

(a) $\frac{1}{\lambda}$ (b) $\frac{1}{7\lambda}$ (c) $\frac{1}{8\lambda}$ (d) $\frac{1}{9\lambda}$

7 Samples of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration constants of A and B, respectively. In which of the following cases, the two samples can simultaneously have the same decay rate at any time?

(a) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$
(b) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$
(c) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$
(d) Initial rate of decay of B is always same

8 The half-life of a radioactive material is 3 h. If the initial amount is 300 g, then after 18 h, it will remain

(a) 4.68 g (b) 46.8 g
(c) 9.375 g (d) 93.75 g

9 The half-life of a radioactive isotope X is 50 yr. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio of 1 : 15 in a sample of a given rock. The age of the rock was estimated to be ➜ CBSE AIPMT 2011

(a) 200 yr (b) 250 yr (c) 100 yr (d) 150 yr

10 The half-life of radioactive element is 600 yr. The fraction of sample that would remain after 3000 yr, is

(a) 1/2 (b) 1/16 (c) 1/8 (d) 1/32

11 A radioactive sample has an initial activity of 50 dpm, 20 min later, the activity is 25 dpm. How many atoms of the radioactive nuclide were there originally?

(a) 20 (b) 80 (c) 1443 (d) 5441

12 A radio isotope X with a half-life 1.4×10^9 yr decays to Y which is stable. A sample of the rock from a cave was found to contain X and Y in the ratio 1 : 7. The age of the rock is ➜ CBSE AIPMT 2014

(a) 1.96×10^9 yr (b) 3.92×10^9 yr
(c) 4.20×10^9 yr (d) 8.40×10^9 yr

13 The half-life of a radioactive isotope X is 20 yr. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio 1 : 7 in a sample of a given rock. The age of the rock is estimated to be ➜ NEET 2013

(a) 40 yr (b) 60 yr (c) 80 yr (d) 100 yr

14 If the half-life of any sample of radioactive substance is 4 days, then the fraction of sample will remain undecayed after 2 days, will be

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (d) $\frac{1}{2}$

15 The half-life of a radioactive substance is 30 min. The time (in minutes) taken between 40% decay and 85% decay of the same radioactive substance is ➜ NEET 2016

(a) 15 (b) 30 (c) 45 (d) 60

16 For a radioactive material, half-life is 10 min. If initially there are 600 number of nuclei, the time taken (in min) for the disintegration of 450 nuclei is ➜ NEET 2018

(a) 30 (b) 10 (c) 20 (d) 15

17 If a radioactive substance decays for time interval equal to its mean life, then the fraction of the substance remaining undecayed, will be

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) e^2 (d) e

18 A neutron causes fission in $_{92}U^{235}$ producing $_{40}Zr^{97}$ and Te^{134}; and some neutrons. The atomic number of Te will be

(a) 50 (b) 51 (c) 52 (d) 53

19 If 200 MeV energy is obtained per fission of $_{92}U^{235}$, then the number of fissions per second to produce 1kW power will be

(a) 1.25×10^{18} (b) 3.2×10^{-8}
(c) 3.125×10^{13} (d) 0.125×10^{13}

20 On fission of one nucleus of U^{235}, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kW. Number of nuclei fissioned per second in the reactor, is

(a) 9.4×10^{16} s^{-1} (b) 2.3×10^{8} s^{-1}
(c) 3.125×10^{16} s^{-1} (d) 4.2×10^{8} s^{-1}

21 10^{14} fissions per second are taking place in a nuclear reactor having efficiency 40%. The energy released per fission is 250 MeV. The power output of the reactor is

(a) 2000 W (b) 4000 W
(c) 1600 W (d) 3200 W

22 Fusion reaction takes place at high temperature, because ➜ CBSE AIPMT 2011

(a) atoms get ionised at high temperature
(b) kinetic energy is high enough to overcome the Coulomb repulsion between nuclei
(c) molecules break up at high temperature
(d) nuclei break up at high temperature

23 If mass of proton = 1.008 amu and mass of neutron = 1.009 amu, then the binding energy per nucleon for ${}_4Be^9$ (mass = 9.012 amu) will be

(a) 0.0672 MeV (b) 0.672 MeV
(c) 6.72 MeV (d) 67.2 MeV

24 The mass of ${}_{18}Ar^{40}$ is 39.9480 amu. Its mass defect will be (Take, m_p = 1.0078 amu and m_n = 1.0087 amu)

(a) 0.3694 amu (b) 0.3318 amu
(c) 0.3480 amu (d) 0.3838 amu

25 The binding energy of neutron in deuterium ${}_1H^2$ will be (Take, m_p = 1.0078 amu, m_n = 1.0087 amu and m_d = 2.0141 amu)

(a) 2.2344 MeV (b) 4.4688 MeV
(c) 1.1172 MeV (d) 7.8 MeV

26 The energy released in the following β-decay process will be ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}q^0 + \bar{\nu}$

Take, $m_n = 1.6747 \times 10^{-27}$ kg,

$m_p = 1.6725 \times 10^{-27}$ kg and $m_e = 0.00091 \times 10^{-27}$ kg

(a) 0.931 MeV (b) 0.731 MeV
(c) 0.511 MeV (d) 0.271 MeV

27 If the mass defect in a fusion process is 0.3%, then the energy released in the fusion of 1kg of material will be

(a) 2.7×10^{14} J (b) 2.7×10^{-14} J
(c) 2.7×10^{14} erg (d) 2.7×10^{14} eV

28 Heavy stable nuclei have more neutrons than protons. This is because of the fact, that

(a) neutrons are heavier than protons
(b) electrostatic force between protons is repulsive
(c) neutrons decay into protons through β-decay
(d) nuclear forces between neutrons is weaker than that between protons

29 If all the atoms of 1kg of deuterium undergo nuclear fusion, then the amount of energy released will be

(a) 6×10^{27} cal (b) 2×10^7 kWh
(c) 56.9×10^{13} J (d) 8×10^{23} MeV

30 The mass of uranium required per day to generate 1MW power from the fission of ${}_{92}U^{235}$, will be

(a) 1.05 g (b) 2.05 g (c) 3.05 g (d) 4.05 g

31 The binding energy per nucleon of ${}^7_3Li$ and ${}^4_2He$ nuclei are 5.60 MeV and 7.06 MeV, respectively. In the nuclear reaction ${}^7_3Li + {}^1_1H \rightarrow {}^4_2He + {}^4_2He + Q$, the value of energy Q released is ➜ CBSE AIPMT 2014

(a) 19.6 MeV (b) –2.4 MeV (c) 8.4 MeV (d) 17.3 MeV

32 A certain mass of hydrogen is changed to helium by the process of fusion. The mass defect in fusion reaction is 0.02866 u. The energy liberated per u, is

(Take, 1u = 931 MeV) ➜ NEET 2013

(a) 2.67 MeV (b) 26.7 MeV
(c) 6.675 MeV (d) 13.35 MeV

33 Consider the nuclear reaction $X^{200} \longrightarrow A^{110} + B^{80}$. If the binding energy per nucleon for X, A and B are 7.4 MeV, 8.2 MeV and 8.1 MeV, respectively. Then, the energy released in the reaction is

(a) 70 MeV (b) 200 MeV (c) 190 MeV (d) 10 MeV

34 The mass of a ${}^7_3Li$ nucleus is 0.042u less than the sum of the masses of all its nucleons. The binding energy per nucleon of ${}^7_3Li$ nucleus is nearly ➜ CBSE AIPMT 2010

(a) 46 MeV (b) 5.6 MeV
(c) 3.9 MeV (d) 23 MeV

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The activity of a radioactive sample is measured as 9750 counts/min at $t = 0$ and as 975 counts/min at $t = 5$ min. The decay constant is approximately

(a) 0.922/min (b) 0.691/min (c) 0.461/min (d) 0.230/min

2 A mixture consists of two radioactive materials A_1 and A_2 with half-lives of 20 s and 10 s, respectively. Initially, the mixture has 40 g of A_1 and 160 g of A_2. The amount of the two in the mixture will become equal after ➜ CBSE AIPMT 2012

(a) 60 s (b) 80 s (c) 20 s (d) 40 s

3 If 10% of a radioactive substance decays in every 5 yr, then the percentage of the substance that will have decayed in 20 yr, will be

(a) 40% (b) 50% (c) 65.6% (d) 34.4%

4 The activity of a radioactive sample is measured as N_e counts per minute at $t = 0$ and N_0/e counts per minute at $t = 5$ min. The time (in minute) at which the activity reduces to half its value, is ➜ CBSE AIPMT 2010

(a) $\log_e 2/5$ (b) $\frac{5}{\log_e 2}$ (c) $5 \log_{10} 2$ (d) $5 \log_e 2$

5 Half-life of a radioactive substance is 20 min. The time between 20% and 80% decay will be

(a) 20 min (b) 40 min (c) 80 min (d) 60 min

6 The number N of nuclei of a radioactive element X, at time t, if at time $t = 0$, the element has N_0 number of nuclei. Nuclei of the element X is being produced at a constant rate α and the element has a decay constant λ, is

(a) $N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}$ (b) $N = \frac{\lambda}{\alpha}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}$

(c) $N = \frac{\alpha}{\lambda}(e^{-\lambda t} - e) + N_0$ (d) $N = \frac{\lambda}{\alpha}(e^{-\lambda t} - e) + N_0$

7 The binding energy of deuteron is 2.2 MeV and that of ${}_2^4\text{He}$, is 28 MeV. If two deuterons are fused to form one ${}_2^4\text{He}$, then the energy released is

(a) 25.8 MeV (b) 23.6 MeV (c) 19.2 MeV (d) 30.2 MeV

8 A nuclear explosive is designed to deliver 1 MW power in the form of heat energy. If the explosion is designed with a nuclear fuel consisting of U^{235} to run a reactor at this power level for one year, then the amount of fuel needed is (Take, energy per fission is 200 MeV)

(a) 1 kg (b) 0.01 kg
(c) 3.84 kg (d) 0.384 kg

9 A uranium reactor ${}_{92}U^{235}$ takes 30 days to use up 2 kg of fuel, each fission gives 185 MeV of usable energy, then power output is

(a) 32.23 MW
(b) 22.28 MW
(c) 58.46 MW
(d) None of these

10 A nucleus of uranium decays at rest into nuclei of thorium and helium. Then, ➜ CBSE AIPMT 2015

(a) the helium nucleus has more kinetic energy than the thorium nucleus
(b) the helium nucleus has less momentum than the thorium nucleus
(c) the helium nucleus has more momentum than the thorium nucleus
(d) the helium nucleus has less kinetic energy than the thorium nucleus

11 In the nuclear decay given below

$${}_Z^AX \longrightarrow {}_{Z+1}^{A}Y \longrightarrow {}_{Z-1}^{A-4}B^* \longrightarrow {}_{Z-1}^{A-4}B,$$

the particles emitted in the sequence are

➜ CBSE AIPMT 2009

(a) β, α, γ (b) γ, β, α
(c) β, γ, α (d) α, β, γ

12 In the reaction ${}_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + {}_0^1n$, if the binding energies of ${}_1^2\text{H}$, ${}_1^3\text{H}$ and ${}_2^4\text{He}$ are respectively a, b and c (in MeV), then the energy (in MeV) released in this reaction is

➜ CBSE AIPMT 2005

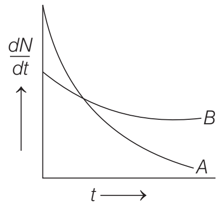
(a) $c + a - b$ (b) $c - a - b$
(c) $a + b + c$ (d) $a + b - c$

13 The mass of proton is 1.0073 u and that of neutron is 1.0087 u (u = atomic mass unit). The binding energy of ${}_2\text{He}^4$ is (mass of helium nucleus = 4.0015 u)

➜ CBSE AIPMT 2003

(a) 28.4 MeV (b) 0.061 u
(c) 0.0305 J (d) 0.0305 erg

14 The variation of decay rate of two radioactive samples A and B with time is shown in figure.



Which of the following statement(s) is/are true?

(a) Decay constant of A is greater than that of B, hence A always decays faster than B
(b) Decay constant of B is greater than that of A, but its decay rate is always smaller than that of A
(c) Decay constant of A is greater than that of B, but it does not always decay faster than B
(d) Decay constant of B is same as that of A, but still its decay rate becomes equal to that of A at a later instant

ANSWERS

SESSION 1	1 (d)	2 (b)	3 (b)	4 (b)	5 (d)	6 (b)	7 (b)	8 (a)	9 (a)	10 (d)
	11 (c)	12 (c)	13 (b)	14 (b)	15 (d)	16 (c)	17 (a)	18 (c)	19 (c)	20 (c)
	21 (c)	22 (b)	23 (c)	24 (d)	25 (c)	26 (b)	27 (a)	28 (b)	29 (c)	30 (a)
	31 (d)	32 (c)	33 (a)	34 (b)						
SESSION 2	1 (c)	2 (d)	3 (d)	4 (d)	5 (b)	6 (a)	7 (b)	8 (d)	9 (c)	10 (a)
	11 (a)	12 (b)	13 (a)	14 (c)						

Hints and Explanations

SESSION 1

1 Total energy produced in the interaction,

$$7E = 1.02 + 1.02 + 0.51 \times 2 = 3.06 \text{ MeV}$$

$\therefore$ Energy of each photon

$$= \frac{3.06}{2} = 1.53 \text{ MeV}$$

2 As an α-particle carries two units of positive charge, a β-particle carries one unit of negative charge and γ-particle carries no charge, therefore electronic energy levels of the atom change for α and β-decay, but not for γ-decay.

3 $A = 222 - 4 = 218$ and $Z = 86 - 2 = 84$

At $Z = 84$ there is polonium.

4 $${}^{m}_{n}X \xrightarrow{\alpha} {}_{n-2}X^{m-4} \xrightarrow{2\beta} {}_{n}X^{m-4}$$

5 Let the radioactive substance be ${}^{A}_{Z}X$.

Radioactive transition is given by

$$ {}^{A}_{Z}X \xrightarrow{-\alpha} {}^{A-4}_{Z-2}X \xrightarrow{-2\beta} {}^{A-4}_{Z}X$$

The atoms of element having same atomic number, but different mass numbers are called isotopes.

So, ${}^{A}_{Z}X$ and ${}^{A-4}_{Z}X$ are isotopes.

6 Let initial number of nuclei in A and B is N_0.

Number of nuclei of A after time t is

$$N_A = N_0 e^{-8\lambda t} \quad \text{...(i)}$$

Similarly, number of nuclei of B after time t is

$$N_B = N_0 e^{-\lambda t} \quad \text{...(ii)}$$

It is given that,

$$\frac{N_A}{N_B} = \frac{1}{e} \quad [\because N_B > N_A]$$

Now, from Eqs. (i) and (ii), we get

$$\frac{e^{-8\lambda t}}{e^{-\lambda t}} = \frac{1}{e}$$

Rearranging

$$\Rightarrow \quad e^{-1} = e^{-7\lambda t} \quad \Rightarrow \quad 7\lambda t = 1$$

$$\Rightarrow \quad \text{Time, } t = \frac{1}{7\lambda}$$

7 The two samples of two radioactive nuclides A and B can simultaneously have the same decay rate at any time, if initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$.

8 Number of half-lives, $n = \frac{t}{T} = \frac{18}{3} = 6$

Amount remained after n half-lives,

$$N = N_0 = \left(\frac{1}{2}\right)^n$$

Given, $\quad N_0 = 300$ g

$$\therefore N = 300\left(\frac{1}{2}\right)^6 = 300 \times \frac{1}{64} = 4.68 \text{ g}$$

9 We know that, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{T_{1/2}}$

$$\Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{t/50} \Rightarrow t = 4 \times 50$$

$$t = 200 \text{ yr}$$

10 We know that, $n = \frac{1}{T}$

Given, $t = 3000$ yr,

$$T = 600 \text{ yr} \quad \Rightarrow \quad n = \frac{3000}{600} = 5$$

Then, $\quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

11 Rate decreases from 50 to 25 dpm in 20 min, hence half-life is 20 min.

$$r_0 = \lambda N_0 = \frac{0.693}{t_{1/2}} \times N_0$$

$$\Rightarrow N_0 = \frac{r_0 \times t_{1/2}}{0.693} = \frac{50 \times 20}{0.693} = 1443$$

12 $X : Y = 1 : 7$

$X : (X + Y) = 1 : 8 = 1 : 2^3 \Rightarrow 3$ half-life

$$\therefore \quad \Delta T = 3 \times 1.4 \times 10^9 \text{ yr} = 4.20 \times 10^9 \text{ yr}$$

13 As, $\quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

As, $\quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Number of half-lives $= 3 \Rightarrow T = 20$ yr

$$\therefore \quad T = \frac{t}{n}$$

or $\quad t = T \times n = 20 \times 3 = 60$ yr

14 From Rutherford-Soddy's law, the fraction left after n half-lives is

$$N = N_0 \left(\frac{1}{2}\right)^n$$

where, n is number of half-lives.

$$n = \frac{\text{Time}(t)}{\text{Half - life}(T_{1/2})} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^{1/2} \Rightarrow \frac{N}{N_0} = \frac{1}{\sqrt{2}}$$

15 Given, $\quad N_1 = 0.6\, N_0 \quad (\because 40\% \text{ decay})$

$\quad N_2 = 0.15\, N_0 \quad (\because 85\% \text{ decay})$

Putting these in the formula,

$$\frac{N_2}{N_1} = \frac{0.15\, N_0}{0.6\, N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

So, two half-life periods has passed.

Thus, time taken $= 2 \times T_{1/2} = 2 \times 30 = 60$ min

16 $\therefore$ Number of nuclei left undecayed,

$$N = N_0 - N' = 600 - 450 = 150$$

Half-life, $T_{1/2} = 10$ min

As, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{T_{1/2}}$

Substituting the given values, we get

$$\frac{150}{600} = \left(\frac{1}{2}\right)^{t/10} \text{ or } \frac{1}{4} = \left(\frac{1}{2}\right)^{t/10}$$

$$\text{or } \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/10} \text{ or } \frac{t}{10} = 2$$

$$\Rightarrow \quad t = 20 \text{ min}$$

17 $N = N_0 e^{-\lambda t}$, if $t = \frac{1}{\lambda} = T_m$,

then $\frac{N}{N_0} = e^{-\lambda \cdot \frac{1}{\lambda}} = \frac{1}{e}$

18 $${}_{92}U^{235} \longrightarrow {}_{40}Zr^{97} + Te^{134} + x_0 n^1$$

According to the law of conservation of charge,

$$92 = 40 + Z \text{ or } Z = 52$$

19 Number of fissions per second

$$= \frac{\text{Required rate of emission of energy}}{\text{Energy emitted per fission}}$$

$$= \frac{10^3}{200 \times 10^6 \times 1.6 \times 10^{-19}}$$

$$= 3.125 \times 10^{13} \text{ fission/s}$$

20 Power received $= 1000$ kW

$= 1000 \times 1000 \text{ W} = 10^6 \text{ Js}^{-1}$

$1 \text{ MeV} = 1.6 \times 10^{-13} \text{J}$

$$\therefore P = \frac{10^6}{1.6 \times 10^{-13}} = 6.25 \times 10^{18} \text{ MeVs}^{-1}$$

Number of nuclei fissioned (per second)

is $= \frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16} \text{s}^{-1}$

21 Total energy released per second

$$= 250 \times 1.6 \times 10^{-13} \times 10^{14} = 4000 \text{ J}$$

Power output $= 4000 \times \frac{40}{100} = 1600$ W

22 Fusion reaction takes place at high temperature, because kinetic energy is high enough to overcome the Coulomb repulsion between nuclei.

23 Mass defect,

$\Delta m = (4 \times 1.008 + 5 \times 1.009) - 9.012$
$= 4.032 + 5.045 - 9.012$
$= 9.077 - 9.012 = 0.065\,\text{amu}$

$\therefore \frac{BE}{A} = \frac{0.065 \times 931}{9} = 6.72\,\text{MeV}$

24 $\Delta m = Zm_p + (A - Z)m_n - M(Z, A)$

$= 18 \times 1.0078 + (40 - 18) \times 1.0087 - 39.9480$
$= 18.1404 + 22.1914 - 39.9480$
$= 0.3838\,\text{amu}$

25 $BE = \{(m_p + m_n) - m_d\} \times 931\,\text{MeV}$

$= \{(1.0078 + 1.0087) - 2.014\} \times 931$
$= 2.2344\ \text{MeV}$

$\therefore \frac{BE}{A} = 1.1172\,\text{MeV}$

26 Mass defect,

$\Delta m = (1.6747 - 1.6725 - 0.0091) \times 10^{-27}$
$= 0.0012 \times 10^{-27}\,\text{kg}$

$\therefore \Delta E = \frac{0.0012 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-13}}$
$= 0.731\,\text{MeV}$

27 $\therefore \Delta m = \frac{0.3}{100} \times 1 = 0.003\,\text{kg}$

$\Delta E = \Delta mc^2$

$\Rightarrow \quad = 0.003 \times (3 \times 10^8)^2$
$= 2.7 \times 10^{14}\,\text{J}$

28 Heavy nuclei, which are stable contain more neutrons than protons in their nuclei. This is because, electrostatic force between protons is repulsive, which may reduce stability.

29 Fusion reaction of deuterium,

${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + 23.6\,\text{MeV}$

Energy released in 1 kg of deuterium

$= \frac{6.02 \times 10^{23} \times 10^3 \times 23.6 \times 1.6 \times 10^{-13}}{2 \times 2}$

$= 56.9 \times 10^{13}\,\text{J}$

30 Energy per day $= 10^6 \times 24 \times 60 \times 60\,\text{J}$

Energy received per fission $= 200\,\text{MeV}$
$= 3.2 \times 10^{-11}\,\text{J}$

Mass of U^{235} per day

$= \frac{235 \times 1.67 \times 10^{-27} \times 10^6 \times 24 \times 60 \times 60}{3.2 \times 10^{-11}}$

$= 1.05 \times 10^{-3}\,\text{kg} = 1.05\,\text{g}$

31 $Q = 2\,(\text{BE of He}) - (\text{BE of Li})$

$= 2 \times (4 \times 7.06) - (7 \times 5.60)$
$= 56.48 - 39.2 = 17.3\,\text{MeV}$

32 Here, $\Delta m = 0.02866\,\text{u}$

$\therefore$ Energy liberated

$= \frac{0.02866 \times 931}{4} = \frac{26.7}{4}\,\text{MeV}$
$= 6.675\,\text{MeV}$

33 For X, energy $= 200 \times 7.4 = 1480\,\text{MeV}$

For A, energy $= 110 \times 8.2 = 902\,\text{MeV}$
For B, energy $= 80 \times 8.1 = 648\,\text{MeV}$
Therefore, energy released
$= (902 + 648) - 1480$
$= 1550 - 1480 = 70\,\text{MeV}$

34 If $m = 1\,\text{u}, c = 3 \times 10^8\,\text{ms}^{-1}$, then

$E = 931\ \text{MeV}$, i.e. $1\,\text{u} = 931\,\text{MeV}$

Binding energy $= 0.042 \times 931$
$= 39.10\,\text{MeV}$

$\therefore$ Binding energy per nucleon

$= \frac{39.10}{7} = 5.58 \approx 5.6\,\text{MeV}$

SESSION 2

1 According to law of radioactivity,

$$\frac{N}{N_0} = e^{-\lambda t} \quad \text{...(i)}$$

$$\Rightarrow \frac{N_0}{N} = e^{\lambda t}$$

On taking logarithm both sides of Eq. (i), we get

$$\log_e\left(\frac{N_0}{N}\right) = \log_e (e^{\lambda t})$$

$$= \lambda t \log_e e = \lambda t$$

As we know that, $\log_e x = 2.3026 \log_{10} x$

Making substitution, we get

$$\lambda = \frac{2.3026 \log_{10}\left(\frac{9750}{975}\right)}{5}$$

$$= \frac{2.3026}{5} \log_{10} 10 = \frac{2.3026}{5}\,\text{min}^{-1}$$

$$= 0.461\,\text{min}^{-1}$$

2 For 40 g amount,

$$40\,\text{g} \xrightarrow[\text{half-life}]{20\,\text{s}} 20\,\text{g} \xrightarrow{20\,\text{s}} 10\,\text{g}$$

For 160 g amont,

$$160\,\text{g} \xrightarrow{10\,\text{s}} 80\,\text{g} \xrightarrow{10\,\text{s}} 40\,\text{g}$$

$$\xrightarrow{10\,\text{s}} 20\,\text{g} \xrightarrow{10\,\text{s}} 10\,\text{g}$$

So, after 40 s, A_1 and A_2 remain same.

3 $\therefore$ $$t = \frac{T \log_{10}\left(\frac{N_0}{N}\right)}{\log_{10} 2}$$

or $$t \propto \log_{10}\left(\frac{N_0}{N}\right)$$

$\therefore$ $$\frac{5}{20} = \frac{\log\left(\frac{100}{90}\right)}{\log\left(\frac{N_0}{N}\right)}$$

or $$\frac{N}{N_0} = \left(\frac{9}{10}\right)^4 = 0.656$$

The fraction of material decayed

$= 1 - 0.656 = 0.344 = 34.4\%$

4 Fraction remains after n half-lives,

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

Given, $N = \frac{N_0}{e}$

$$\Rightarrow \frac{N_0}{eN_0} = \left(\frac{1}{2}\right)^{5/T}$$

$$\Rightarrow \frac{1}{e} = \left(\frac{1}{2}\right)^{5/T}$$

Taking log on both sides, we get

$$\log 1 - \log e = \frac{5}{T} \log \frac{1}{2}$$

$$\Rightarrow -1 = \frac{5}{T}(-\log 2) \Rightarrow T = 5 \log_e 2$$

Now, let t be the time after which acitvity reduces to half

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{t'/5\log_e 2} \Rightarrow t' = 5 \log_e 2$$

5 Half-life of a radioactive substance is 20 min, i.e. $t_{1/2} = 20\,\text{min}$

For 20% decay, we have 80% of the substance left,

$$\frac{80N_0}{100} = N_0 e^{-\lambda t_{20}} \quad \text{...(i)}$$

where, N_0 = initial undecayed substance and t_{20} is the time taken for 20% of the substance left, hence

$$\frac{20N_0}{100} = N_0 e^{-\lambda t_{80}} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$4 = e^{\lambda(t_{80} - t_{20})}$$

$$\ln 4 = \lambda(t_{80} - t_{20})$$

$$2 \ln 2 = \frac{0.693}{t_{1/2}}(t_{80} - t_{20})$$

$$\Rightarrow t_{80} - t_{20} = 2 \times T_{1/2} = 40\,\text{min}$$

6 $\therefore$ $$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \frac{dN}{\alpha - \lambda N} = dt$$

On integrating both sides,

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + A$$

where, A is integration constant.

At $t = 0, N = N_0$

$$\frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = A$$

Equation becomes,

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - \lambda N_0)}{-\lambda}$$

$$\Rightarrow \log_e(\alpha - \lambda N) - \log(\alpha - \lambda N_0) = -\lambda t$$

$$\log_e\left[\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right] = -\lambda t$$

$$\Rightarrow \left[\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right] = e^{-\lambda t}$$

$$\lambda N = \alpha(1 - e^{-\lambda t}) + \lambda N_0 e^{-\lambda t}$$

$$N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}$$

7 The reaction can be written as

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + \text{energy}$$

The energy released in the reaction, is difference of binding energies of daughter and parent nuclei.

Hence, energy released = binding energy of ${}_2He^4 - 2 \times$ binding energy of ${}_1H^2$

$= 28 - 2 \times 2.2 = 23.6$ MeV

8 Energy released per fission is

$E = 200$ MeV

$= 200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11}$ J

Number of fissions to produce 1MW ($= 10^6$ W $= 10^6 \text{Js}^{-1}$) power is

$$= \frac{10^6 \text{Js}^{-1}}{3.2 \times 10^{-11} \text{J}}$$

$= 3.125 \times 10^{16} \text{s}^{-1}$

Total energy required to run a 1 MW reactor for one year, is 10^6Js^{-1} $\times (365 \times 24 \times 60 \times 60) = 3.15 \times 10^{13}$ J

Since, 1 fission (1 atom of U^{235}) produces 3.2×10^{-11} J of energy, total number of U^{235} atoms required is

$$= \frac{3.15 \times 10^{13}}{3.2 \times 10^{-11}} = 9.84 \times 10^{23}$$

Now, 6.02×10^{23} atoms of U^{235} are contained in 235 g of U^{235}

$\therefore$ Mass of U^{235} containing 9.84×10^{23} atoms is

$$M = \frac{235}{6.02 \times 10^{23} \text{atoms}} \times 9.84 \times 10^{23} \text{ atoms}$$

$= 384$ g $= 0.384$ kg

9 Number of atoms in 2 kg fuel (U^{235}) is

$$= \frac{6.02 \times 10^{26}}{235} \times 2 = 5.12 \times 10^{24}$$

The nuclei of these atoms are fissioned in 30 days.

Therefore, fission rate is

$$= \frac{5.12 \times 10^{24}}{(30 \times 24 \times 60 \times 60)} = 1.975 \times 10^{18} \text{s}^{-1}$$

Each fission gives 185 MeV of energy. Hence, energy obtained in 1, i.e. power output is

$P = 185 \text{MeV} \times (1.975 \times 10^{18})$

$= 365.4 \times 10^{18} \text{ MeVs}^{-1}$

But 1MeV $= 1.6 \times 10^{-13}$ J

$\therefore P = (365.4 \times 10^{18} \text{ MeVs}^{-1}) \times (1.6 \times 10^{-13} \text{ J / MeV})$

$= 58.46 \times 10^6 \text{Js}^{-1}$

$= 58.46 \times 10^6$ W $= 58.46$ MW

10 ${}_{92}U^{238} \longrightarrow {}_{92}Th^{238} + {}_2He^4$

According to law of conservation of linear momentum, we have

$|\mathbf{P}_{Th}| = |\mathbf{P}_{He}| = \mathbf{P}$

$\Rightarrow$ As, kinetic energy of an element,

$$KE = \frac{P^2}{2m}$$

where, m is mass of an element.

Thus, $\quad KE \propto \frac{1}{M}$

So, $M_{He} < M_{Th}$

$\Rightarrow \quad K_{He} > K_{Th}$

So, helium nucleus has more kinetic energy than the thorium nucleus.

11 Alpha particles are positively charged particles with charge $+2e$ and mass 4 m. Emission of an α-particle reduces the mass of the radionuclide by 4 and its atomic number by 2. β-particles are negatively charged particles with rest mass as well as charge same as that of electrons. γ-particles carry no charge and mass.

Radioactive transition will be as follows

$${}^A_Z X \longrightarrow {}_{Z+1}{}^{A}Y + \beta^0_{-1}$$

$${}_{Z+1}{}^{A}Y \longrightarrow {}^{A-4}_{Z-1}\beta + \alpha^4_2$$

$${}^{A-4}_{Z+1}\beta \longrightarrow {}^{A-4}_{Z-1}\beta + \gamma^0_0$$

12 Given, binding energy of

$({}^2_1H + {}^3_1H) = a + b$

Binding energy of ${}^4_2He = c$

In a nuclear reaction, the resultant nucleus is more stable than the reactants. Hence, binding energy of ${}^4_2He$ will be more than that of $({}^2_1H + {}^3_1H)$.

Thus, energy released per nucleon

= resultant binding energy

Binding energy of product – Binding energy of reactants

$= c - (a + b) = c - a - b$

13 ${}_2He^4$ contains 2 neutrons and 2 protons.

So, mass of 2 protons

$= 2 \times 1.0073 = 2.0146$ u

So, mass of 2 neutrons

$= 2 \times 1.0087$

$= 2.0174$ u

Total mass of 2 protons and 2 neutrons

$= (2.0146 + 2.0174)$ u $= 4.032$ u

Mass of helium nucleus $= 4.0015$ u

Thus, mass defect is lacking of mass in forming the helium nucleus from 2 protons and 2 neutrons.

$\therefore \Delta m$ = mass defect

$= (4.032 - 4.0015)$ u

$= 0.0305$ u

As we know that, 1u = 931 MeV

Hence, binding energy

$\Delta E = (\Delta m) \times 931 = 0.0305 \times 931$

$= 28.4$ MeV

14 From the given figure, we find that rate of decay of A is faster than that of B. It means, decay constant of A is greater than that of B. However, the two curves intersect at P, beyond P, B decays faster than A. And at P, decay rate of both A and B is the same.